

XUtil

Exponential utility function. This function lets you calculate the exponential utility of e.g., an income. If necessary, you can apply a linear transformation to the input value before calculating the utility. This is useful if you need to convert the income to a different monetary unit, or if you need to subtract your initial investment to get a net income. The linear transformation is controlled by the “b” and the “c” parameters.

If you just want to calculate the utility of the input value, without any transformation, use the default “b” and “c” values, i.e., 1 and 0 respectively.

Assuming that you use the default “b” and “c” values, as well as positive “a” and “d” values, the utility function has the following properties:

- (1) The utility function is increasing.
- (2) The utility function is concave.
- (3) The utility of zero is zero.
- (4) If the input goes to infinity, the utility goes to “a”.
- (5) If the input goes to minus infinity, the utility goes to minus infinity.

If an income increases, you probably expect your utility to increase as well. Thus, in most situations you probably want Property 1 to hold true. The most natural way to ensure this, is to use positive “a”, “b”, and “d” values.

If you are comparing two different investments with equal expected returns, you probably prefer the investment with the smallest uncertainty. This attitude towards risk is known as risk aversion. To include risk aversion in your utility function, you need a concave function. Thus, you want Property 2 to hold true as well. Again the most natural way to ensure this, is to use positive “a”, “b”, and “d” values.

The “d” parameter is usually referred to as the “risk tolerance”. If you use a very high “d” value compared to the size of the input values, the utility function will behave almost like a linear function. If your utility function is almost linear, and you are comparing two investments with equal expected returns, you will be indifferent even if one investment is much more uncertain than the other. If you use a very low “d” value, this means that you are very cautious towards risk.

In general the risk tolerance is a very subjective quantity, and depends on

your current status with respect to money and other resources as well as your risk policy. Typically a large company tends to be more risk tolerant than a small company, simply because a large company can afford to lose more than a small one.

Property 3 is directly related to the “c” parameter, and holds true if and only if “c” is zero. As long as your model is consistent with respect to the “c” values, this parameter is less important when comparing different investments. However, as a convention, one usually assumes Property 3 to hold true for utility functions. Thus, we recommend that you use the “c” parameter e.g., to transform the input value to a net income value.

By Property 4, the exponential utility function is limited above by the “a” value. As long as your model is consistent with respect to the “a” values, this parameter is less important when comparing different investments. A common convention, however, is to let “a” be equal to 1.

Default parameter values:

$$a = 1, b = 1, c = 0, d = 1$$

Example:

$$a = 1, b = 1, c = 0, d = 2$$

$$\text{Input} = 4.2 \Rightarrow \text{Output} = 0.878$$

$$\text{Input} = 8.1 \Rightarrow \text{Output} = 0.983$$